

When embedding a vertex  $v$ , we will classify already embedded vertices as follows:

- A vertex  $w$  is **external**, if it has a back edge to a not-yet-embedded ancestor (“above  $v$ ”), or if it is an articulation point with a subordinate block containing an external vertex. All other vertices are **internal**.
- A vertex  $w$  is **live**, if it has a back edge to  $v$  or if it has a subordinate block containing a live vertex.
- Similarly for blocks (by their roots) and back edges.

**R1:** In each live vertex, we process (in this order):

- 1 back edges to  $v$
- 2 subordinate live internal blocks
- 3 subordinate live external blocks

**R2:** when we enter a subordinate block, we choose the walking direction (in this order):

- 1 to a live internal vertex
- 2 to a live external vertex

If this direction differs from the present one, the subordinate block and all its descendant blocks will be flipped.

# The Boyer-Myrvold algorithm

- 1 If the graph has more than  $3n - 6$  edges  $\Rightarrow$  NOT PLANAR.
- 2 Depth-first search on the graph: *Enter*, *Ancestor*, *LowPoint*.
- 4 We embed vertices  $v$  in order of decreasing *Enters*:
  - 5 Embed tree edges from  $v$  down as trivial blocks (2-cycles):
  - 7 For each child of the vertex  $v$ , we walk around the boundary in both directions and we embed back edges to  $v$ . We follow the rules **R1** and **R2**. We stop when we visit an external vertex.
  - 8 If there remains any non-embedded back edge to  $v \Rightarrow$  NOT PLANAR.

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- 4 We embed vertices  $v$  in order of decreasing *Enters*:
  - 5 Embed tree edges from  $v$  down as trivial blocks (2-cycles):
  - 6 We mark the live subgraph.
  - 7 For each child of the vertex  $v$ , we walk around the boundary in both directions and we embed back edges to  $v$ . We follow the rules **R1** and **R2**. We stop when we visit an external vertex.
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