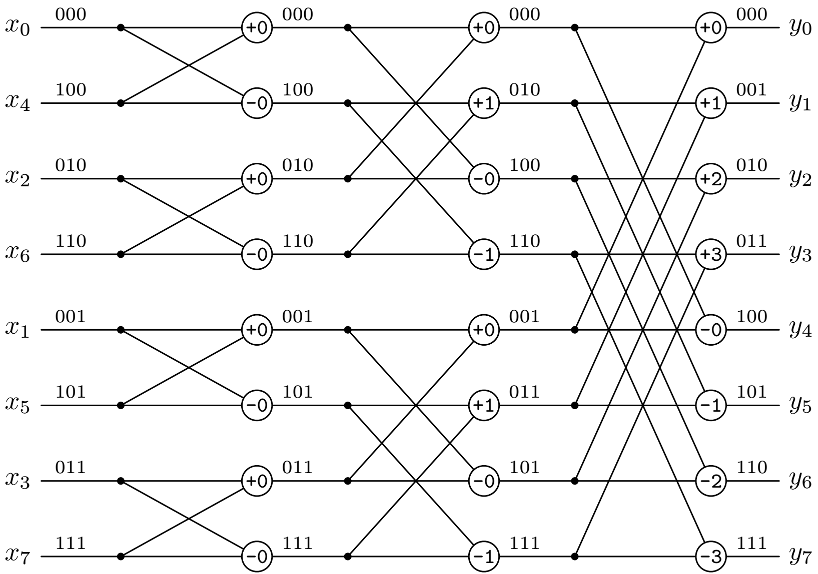
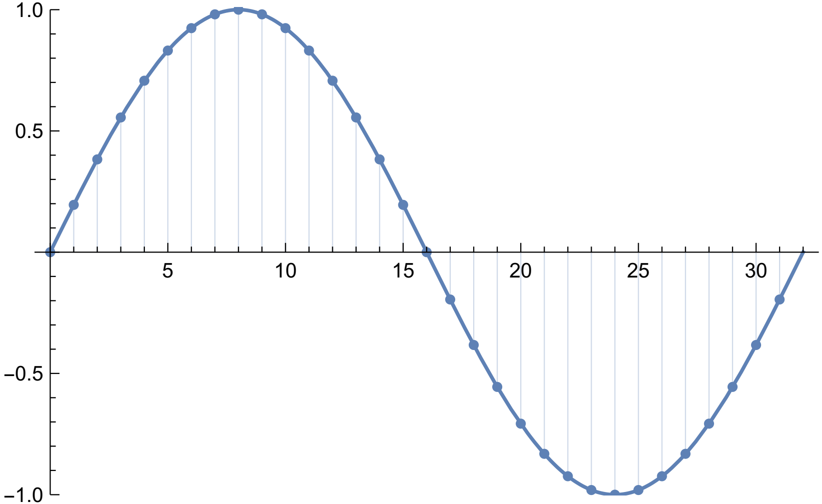
FFT Circuit

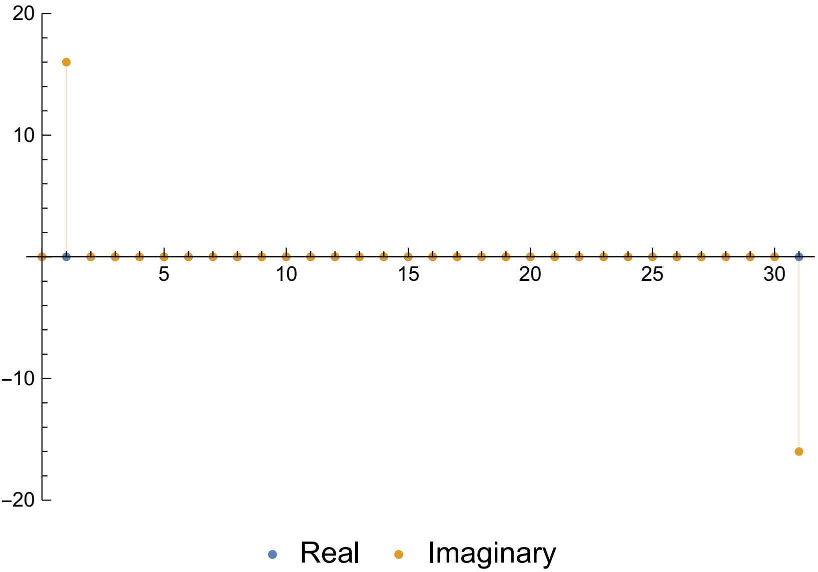


x_0 000 +0 000 +0 000 +0 000 y_0
x_4 100 −0 100 +1 010 +1 001 y_1
x_2 010 +0 010 −0 100 +2 010 y_2
x_6 110 −0 110 −1 110 +3 011 y_3
x_1 001 +0 001 +0 001 −0 100 y_4
x_5 101 −0 101 +1 011 −1 101 y_5
x_3 011 +0 011 −0 101 −2 110 y_6
x_7 111 −0 111 −1 111 −3 111 y_7

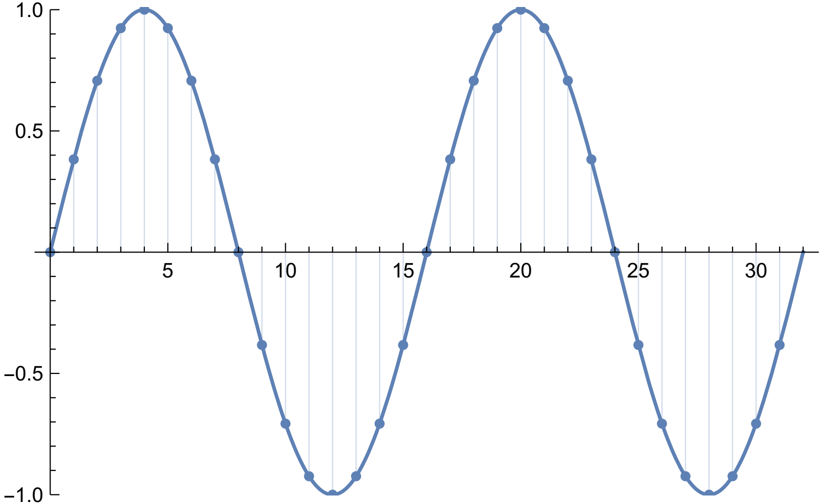
sin(x) sampled in 32 points



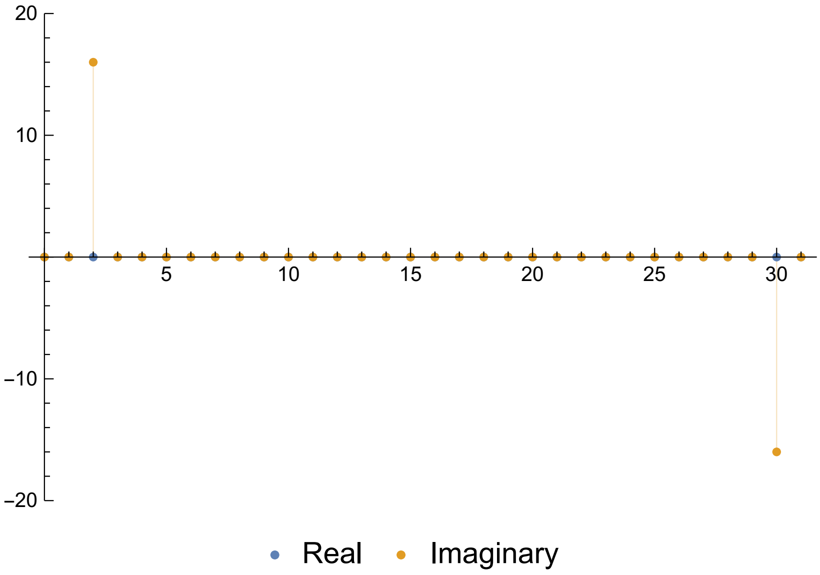
Fourier transform of 32-point $\sin(x)$



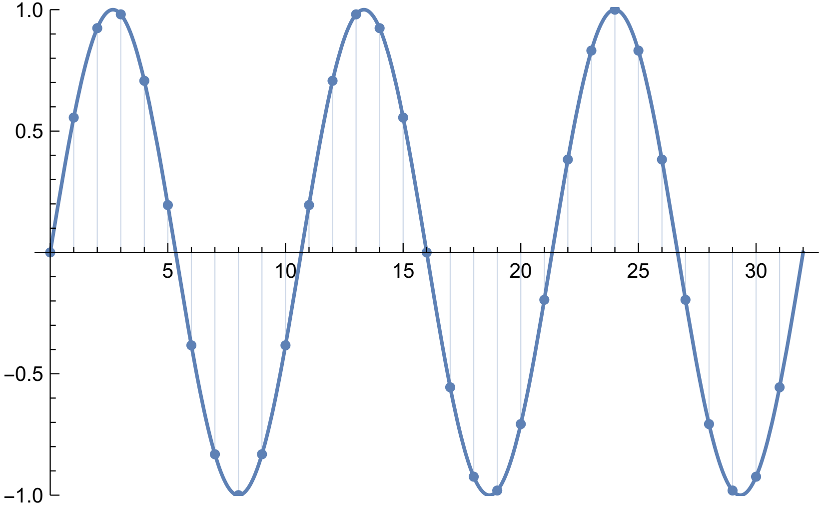
sin(2x) sampled in 32 points



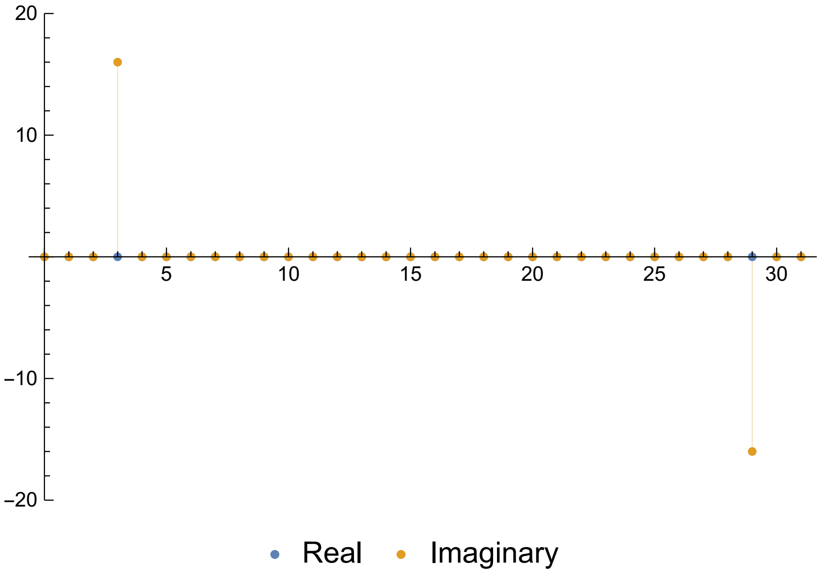
Fourier transform of 32-point $\sin(2x)$



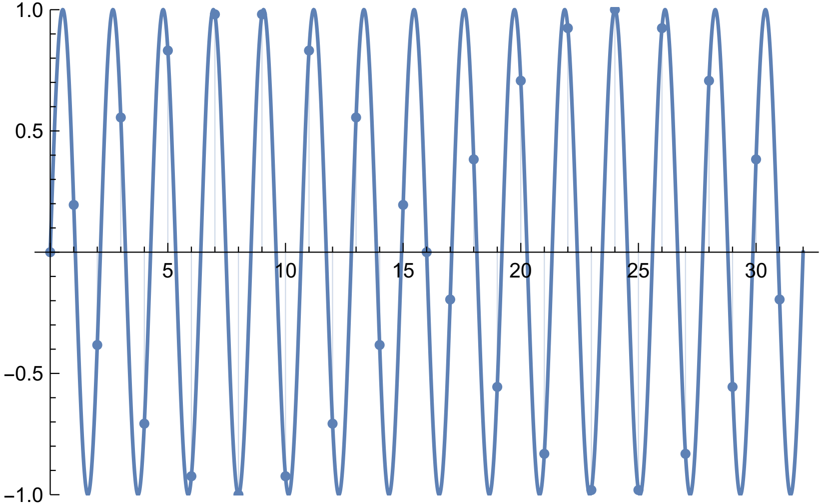
$\sin(3x)$ sampled in 32 points



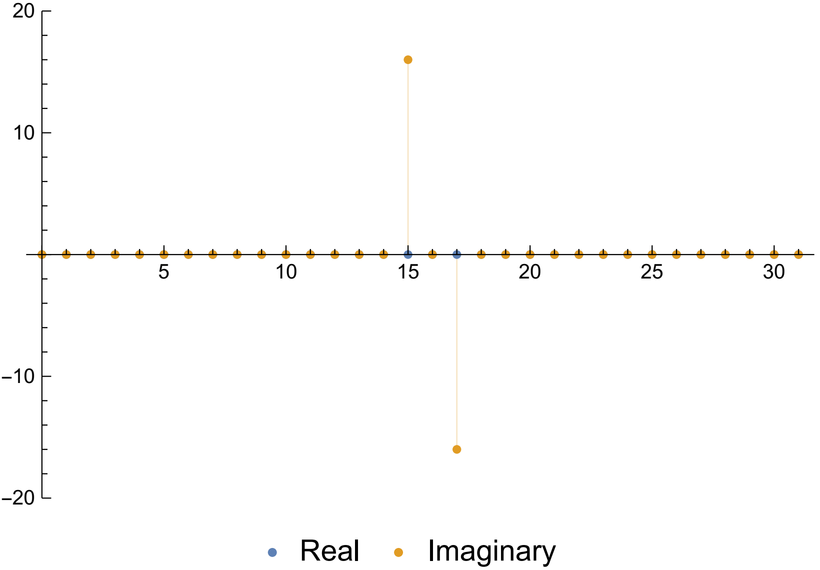
Fourier transform of 32-point $\sin(3x)$



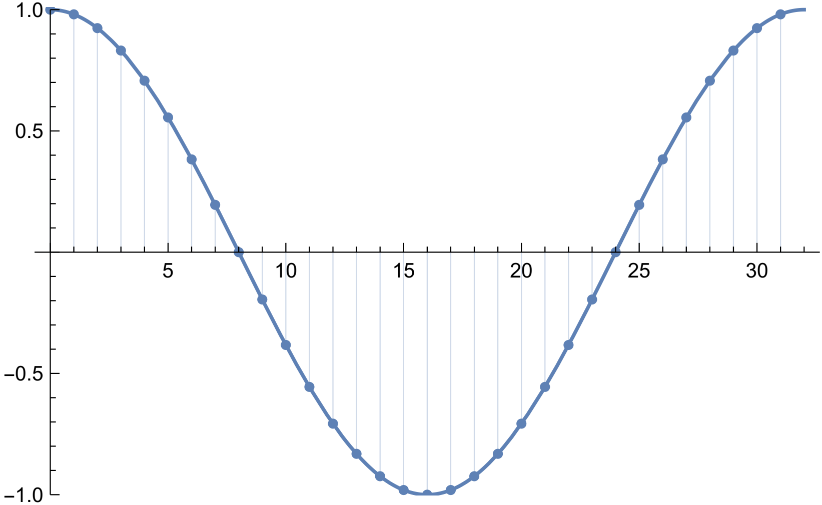
$\sin(15x)$ sampled in 32 points



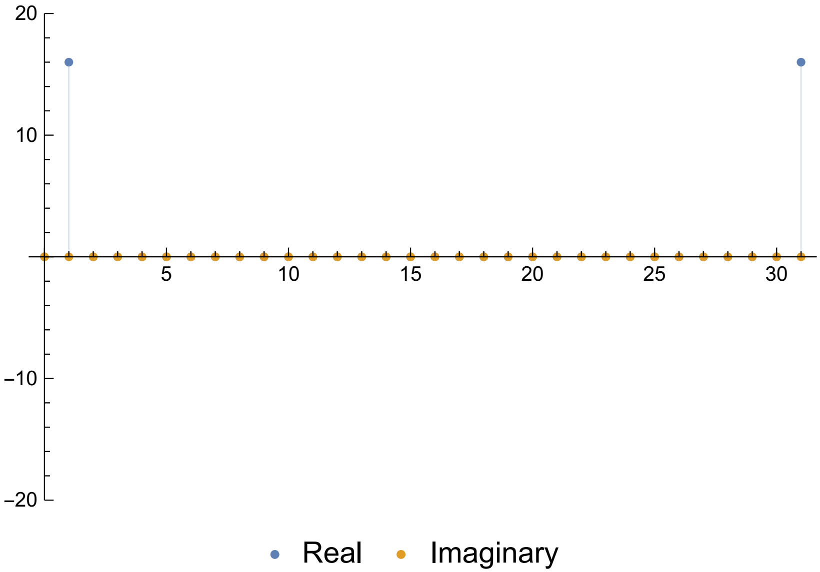
Fourier transform of 32-point $\sin(15x)$



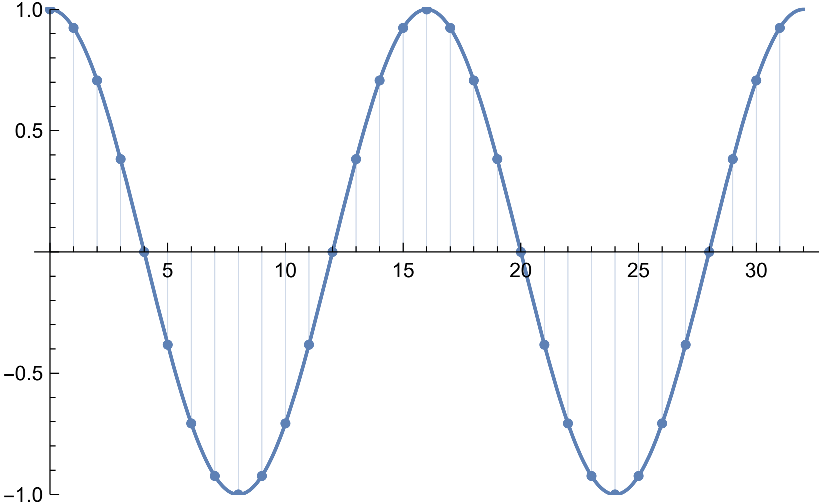
$\cos(x)$ sampled in 32 points



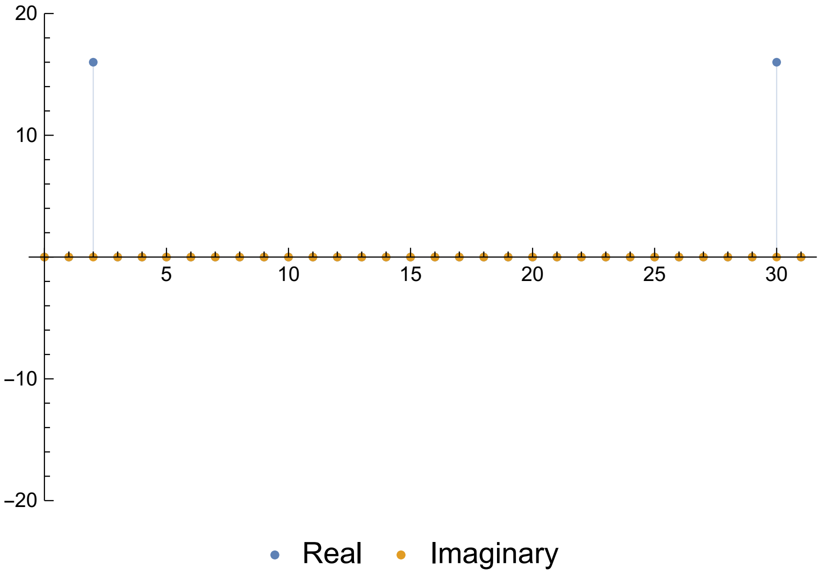
Fourier transform of 32-point $\cos(x)$



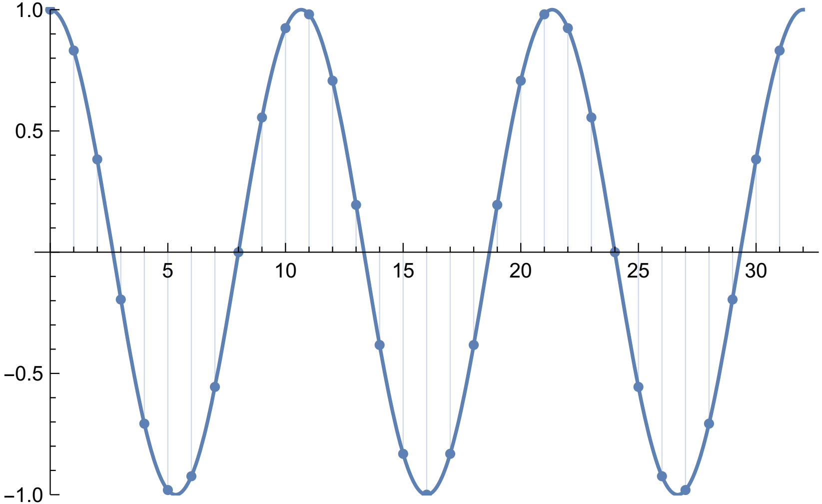
$\cos(2x)$ sampled in 32 points



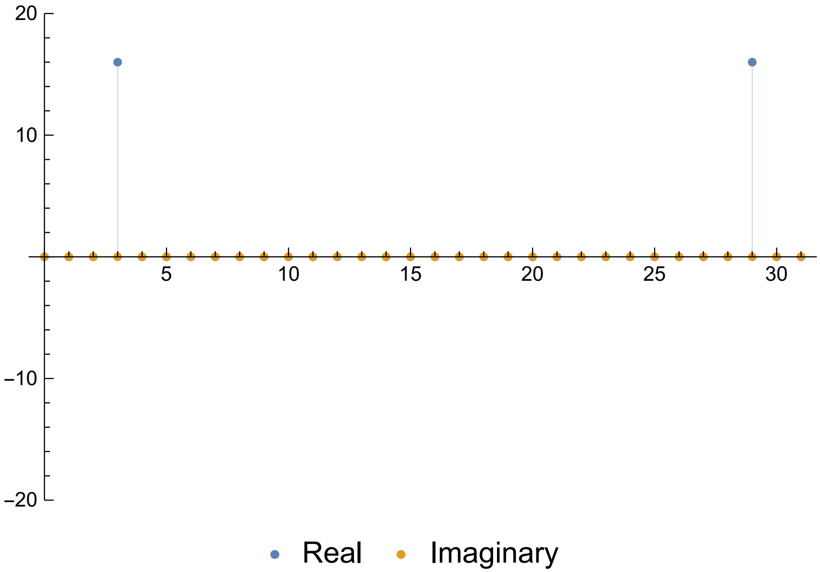
Fourier transform of 32-point $\cos(2x)$



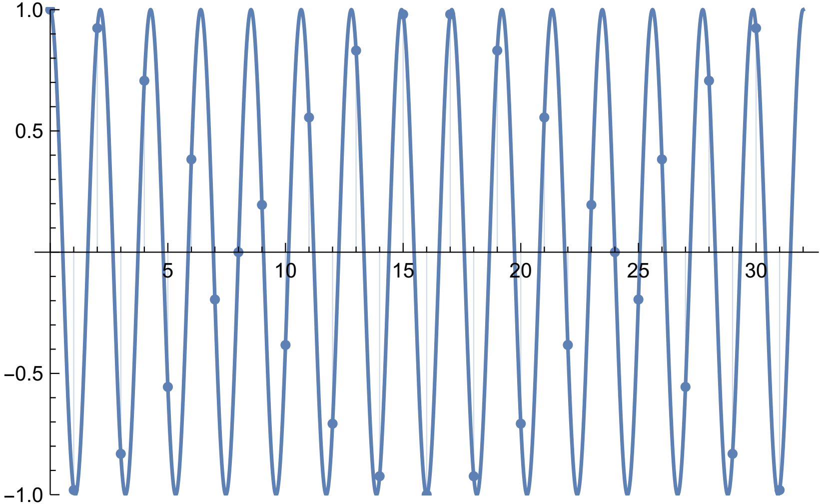
$\cos(3x)$ sampled in 32 points



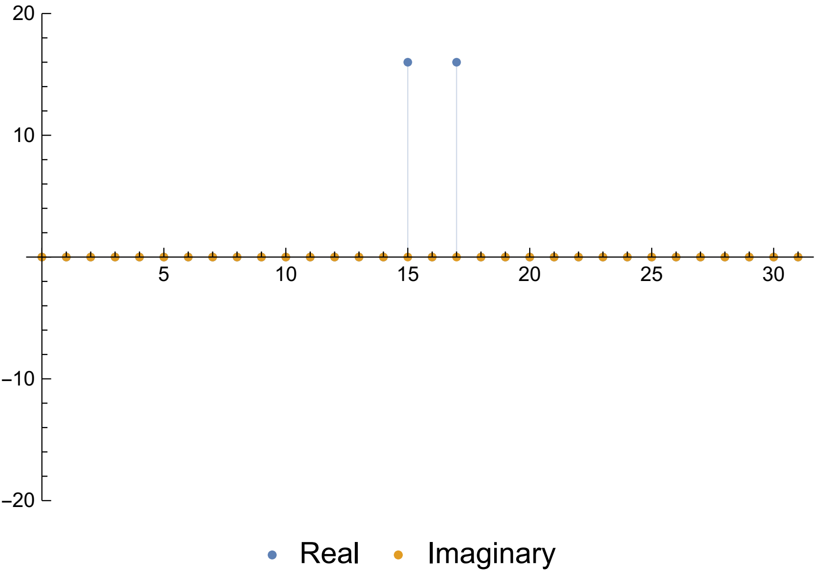
Fourier transform of 32-point $\cos(3x)$



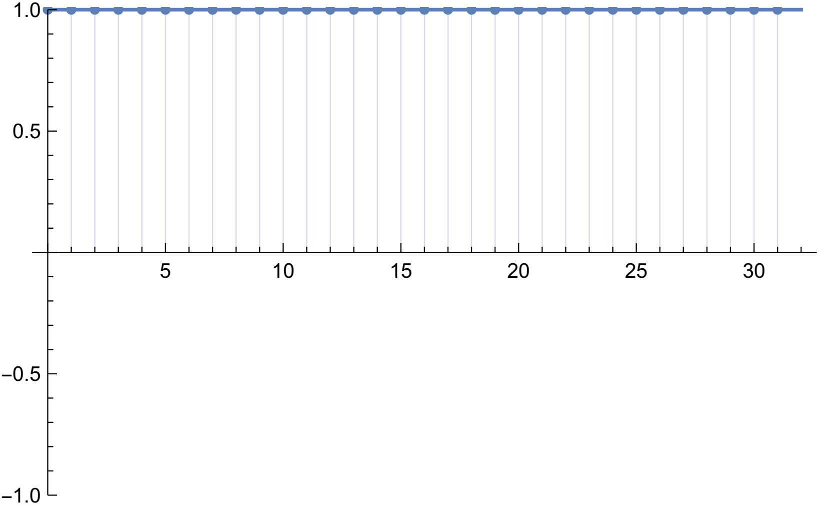
$\cos(15x)$ sampled in 32 points



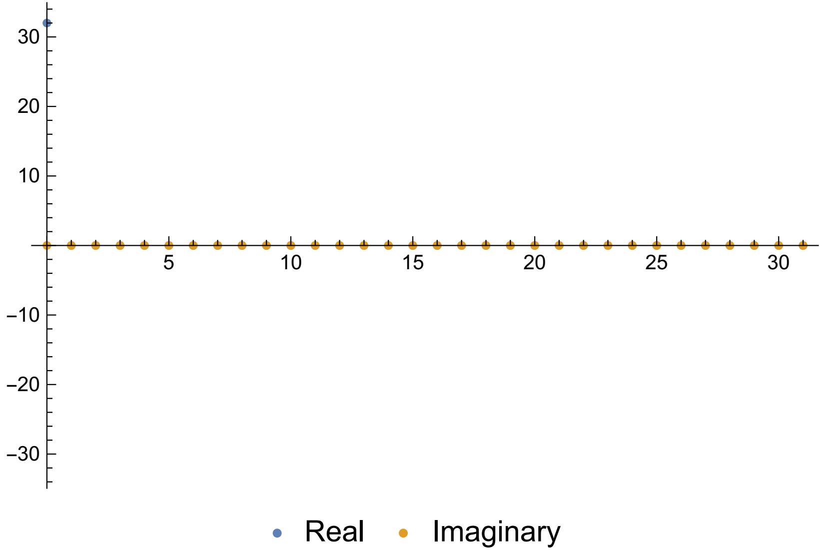
Fourier transform of 32-point $\cos(15x)$



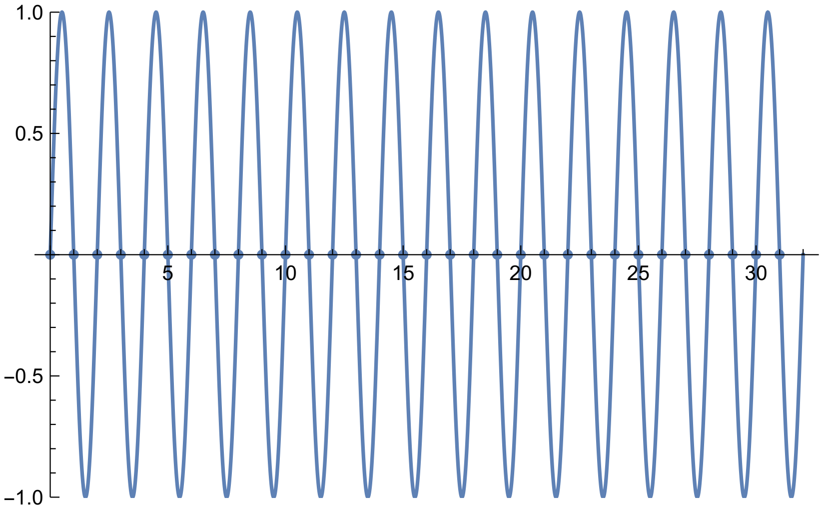
$\cos(0x)$ sampled in 32 points



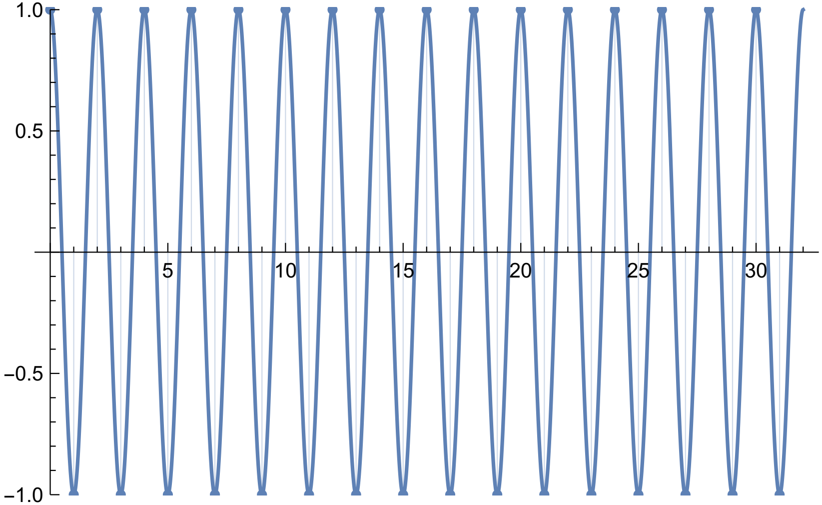
Fourier transform of 32-point $\cos(0x)$



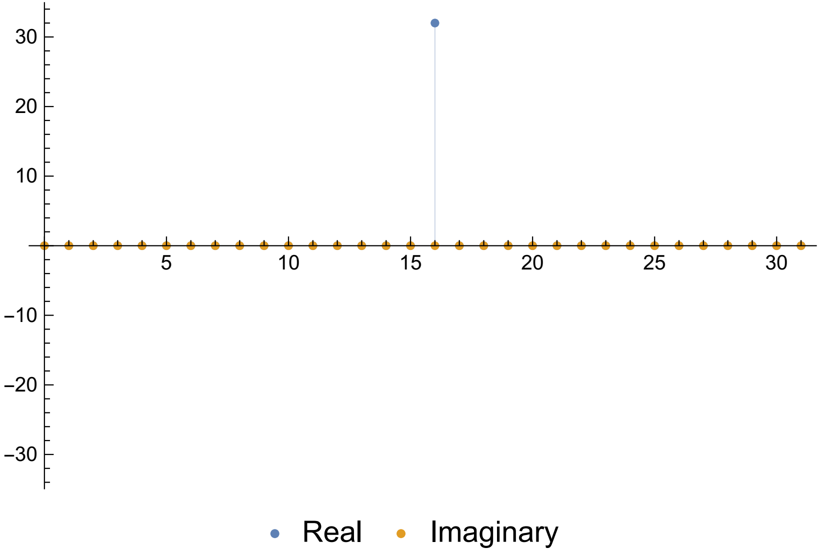
$\sin(16x)$ sampled in 32 points



cos(16x) sampled in 32 points



Fourier transform of 32-point $\cos(16x)$



Real vectors

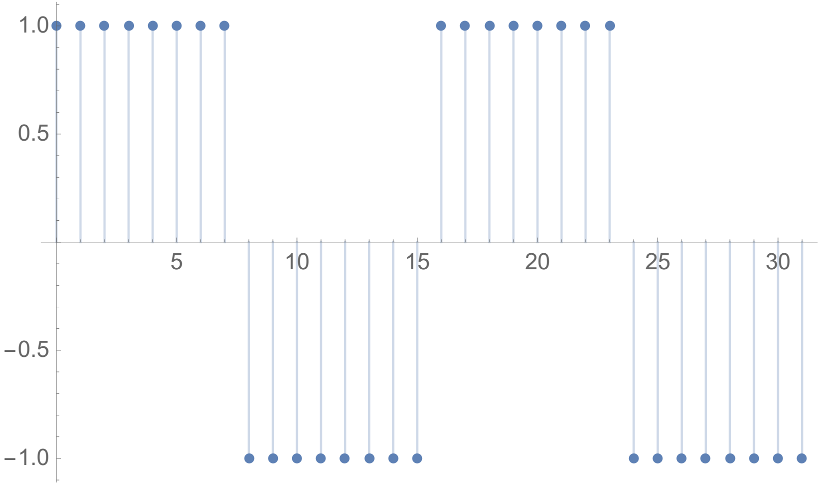
Theorem

Let $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} = \mathcal{F}(\mathbf{x})$. Then $\mathbf{y}_j = \overline{\mathbf{y}_{n-j}}$ for all j.

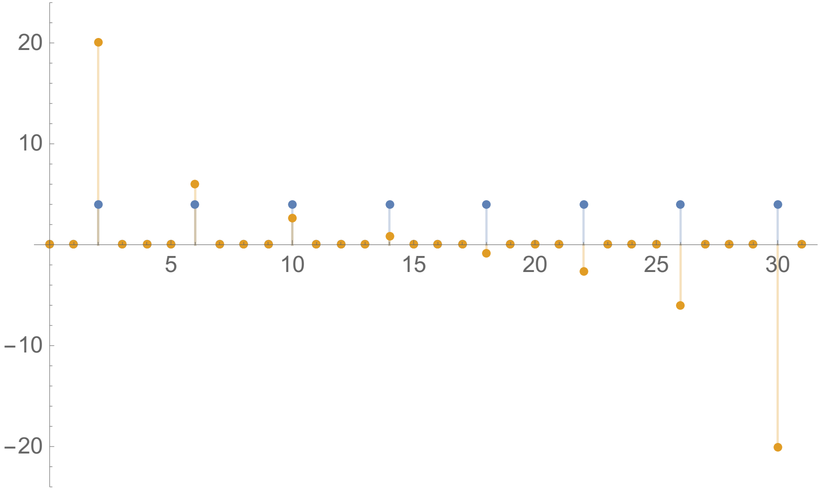
Therefore, each real vector can be written as a linear combination of sampled sines and cosines:

- $\mathrm{Re}(\mathbf{x}_j)$ is the coefficient for $\cos(jx)$ for $j = 1, \ldots, n/2$
- $\mathrm{Im}(\mathbf{x}_j)$ is the coefficient for $\sin(jx)$ for $j = 1, \ldots, n/2 - 1$
- $\mathrm{Re}(\mathbf{x}_0)$ is the additive constant ($\cos(0x)$)
- $\mathrm{Im}(\mathbf{x}_0)$ is always 0
- $\mathrm{Im}(\mathbf{x}_{n/2})$ is always 0

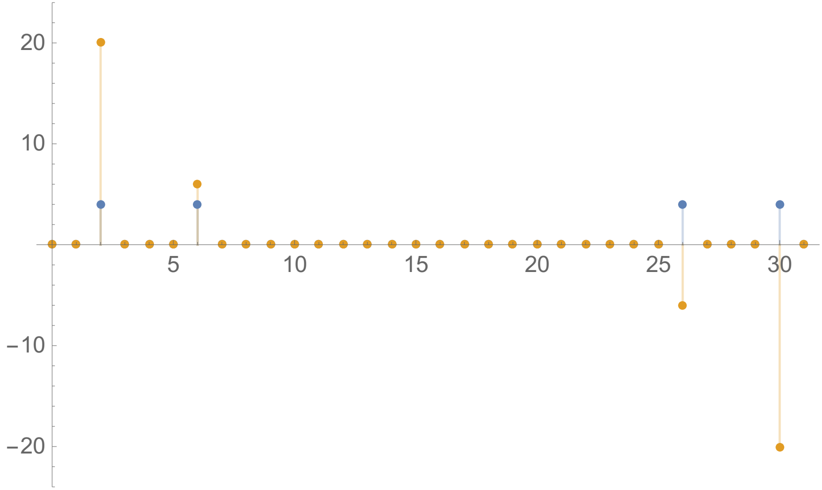
Filtering: a box-shaped signal



Filtering: DFT gives spectrum of the signal



Filtering: DFT without high-frequency components



Filtering: signal without high-frequency components